Insider Trading and Dynamic Informational Efficiency

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Background

The role of private information in financial markets and the informativeness of prices

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There are many theoretical and empirical studies on the effects of insider trading and regulation

- "Insider trading tax"
    - More asymmetric information
    - Less liquidity
- More informative prices

Motivation

What we know

- More informative prices is good
- More liquidity is good

What we don't know much about distributional questions

- Dispersion of private information
- Dispersion of price informativeness
- Dispersion of timing of information revelation

Question

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- We can talk later about the answers to the second and third questions

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Is it better for one trader to have very good private information or for two traders to each have a little private information?

- Shut down the rat race effect
- Does competition matter?

Setup

Consider a little Kyle model.

- There is an asset with common value $v = v_1 + v_2$ with $v_1, v_2 \sim_{iid} \mathcal{N}(0, \sigma_v^2)$
- There is a mass of noise traders that trade for exogenous reasons, $u \sim \mathcal{N}(0, \sigma_u^2)$
- Two traders observe private information and submit trades, x_1 and x_2
- A perfectly competitive market maker observes only the total demand, $z = x_1 + x_2 + u$, and chooses a price
- All players are risk neutral

Informed Trader's Problem

Suppose the market maker and trader 2 are using linear strategies, $p = \lambda z$ and $x_2 = \beta_2 v_2$.

The informed trader can solve their problem pretty quickly

$$\max_{x_1} \mathbb{E}\left[(v_1 + v_2 - p)x_1\right] = \max_{x_1} \mathbb{E}\left[(v_1 + v_2 - \lambda(x_1 + x_2 + u))\, x_1\right]$$
$$= \max_{x_1} v_1 x_1 - \lambda x_1^2$$

Solving,

$$x_1^* = \frac{1}{2\lambda} v_1 \tag{1}$$

Dispersion Doesn't Matter?

Other players

- Informed trader two is exactly the same $x_2 = \frac{1}{2\lambda} v_2$
- Total demand is $\frac{1}{2\lambda}(v_1 + v_2) + u$
- The market maker sets price equal to the expected value of the asset

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What if there was only one informed trader?

- Suppose trader 1 knows both v_1 and v_2 and trader 2 knows nothing
- The problem looks exactly the same in terms of their information $x = \frac{1}{2\lambda}(v_1 + v_2)$
- The demand contains the same information and thus the market maker will respond in the same way

Information Acquisition

Dispersion of private information doesn't matter, but dispersion of opportunities to acquire information does matter

- Unknown asset value $v \sim \mathcal{N}(0, \sigma_v^2)$
- A trader privately observes a signal $v + \epsilon$ with $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon)$ at cost $c(\sigma_\epsilon^2)$
- The trader also observes their private benefit of the asset, $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$
- The market maker observes total demand, $z = x + u$, as before and chooses a price equal to the expected value

Market Maker's Problem

Suppose the insider is using a linear trading strategy, $x = \beta_1(v + \epsilon) + \beta_2\theta$.

Price is equal to expected value

$$\begin{aligned} p &= \mathbb{E}\left[v \mid z\right] \\ &= \mathbb{E}\left[v \mid \beta_1(v+\epsilon) + \beta_2\theta + u\right] \\ &= \frac{Cov(v,z)}{V[z]} z \\ &= \frac{\beta_1\sigma_v^2}{\beta_1\sigma_v^2 + \beta_1\sigma_\epsilon^2 + \beta_2\sigma_\theta^2 + \sigma_u^2} z \\ &= \lambda z \end{aligned}$$

Note that λ is a decreasing function of σ_ϵ.

Insider's Problem

The informed trader maximizes expected profits after observing the signal, $\tilde{v} = v + \epsilon$.

$$\begin{aligned} & \max_x \quad \mathbb{E}\left[(v + \theta - p)\, x\right] \\ = & \max_x \quad \mathbb{E}\left[(v + \theta - \lambda(x + u))\, x\right] \\ = & \max_x \quad \left(\frac{\sigma_v}{\sigma_v + \sigma_\epsilon}\tilde{v} + \theta\right) x - \lambda x^2 \end{aligned}$$

This gives a linear trading strategy.

$$x^* = \frac{1}{2\lambda}\left(\frac{\sigma_v}{\sigma_v + \sigma_\epsilon}\tilde{v} + \theta\right) \qquad (2)$$

Profits

To find the optimal information acquisition strategy, we need to compute the expected profit of receiving signal with noise σ_ϵ.

$$\begin{aligned}\pi &= \mathbb{E}\left[(v+\theta-p)\,x^*\right]\\ &= \frac{1}{2\lambda}\mathbb{E}\left[(v+\theta)\left(\frac{\sigma_v}{\sigma_v+\sigma_\epsilon}\tilde{v}+\theta\right)-\frac{1}{2}\left(\frac{\sigma_v}{\sigma_v+\sigma_\epsilon}\tilde{v}+\theta\right)^2\right]\\ &= \frac{1}{4\lambda}\left(\theta^2 \;+\; \frac{\sigma_v^3}{2}\frac{\sigma_v+2\sigma_\epsilon\sigma_v}{(\sigma_v+\sigma_\epsilon)^2}\right)\end{aligned}$$

Results

There are three terms in the information acquisition problem.

$$\max_{\sigma_\epsilon} \quad \frac{\theta^2}{4\lambda} + w(\sigma_\epsilon) - c(\sigma_\epsilon) \tag{3}$$

- $c(\sigma_\epsilon)$ is the cost of acquiring information
- $w(\sigma_\epsilon)$ is the additional trading profit from having the information
- $\frac{\theta^2}{4\lambda}$ is the liquidity cost of more information

The Information Acquisition Arms Race

Every trader chooses how much information to acquire

- Acquiring more information leads to lower liquidity
- Lower liquidity is a negative externality on everyone
- There is an over-acquisition of information

Conclusion

More dispersion of information among traders

- Doesn't matter

More dispersion of information acquisition opportunities

- Lower welfare among traders
- More informative prices

The same thing would work in any kind of model.