Ratings and Reputation

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Abstract

I present a model of universities to explain patterns of grade inflation that have been observed. I model the university as an information designer hired to evaluate students for employment purposes. I find both pooling and separating equilibria. I show how the university's desire to build or maintain their own reputation interacts with their desire for the success of their students in forming the optimal grading scheme. In every equilibrium grade inflation is concentrated at the high quality universities as observed in the data. I also show how the observed patterns of grade inflation are not likely to be driven by heterogeneous students selecting into quality universities.

1 Introduction

Grade inflation is now a well documented phenomenon as well as a topic being discussed at major universities around the world. At today's colleges an A just doesn't mean what it used to. For decades the average GPA at universities as been steadily rising. What is more, grade inflation is not happening uniformly for all universities. Grade inflation has been more significant in the most prestigious universities. Today the average GPA at Harvard, Stanford, and Yale is over 3.6. While Middle-of-Nowhere Tech is still handing out C's regularly, the most common grade at Harvard has become an A. If the most common grade handed out is an A, there isn't much ability for the school to distinguish its best students. If the role of grades is to evaluate students and separate them for potential employers, it would seem a more uniform grading scheme would be better served.

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Similar patterns have been seen for credit rating agencies of corporate bonds. The ratings of the most established agencies have grown less informative over time, while the less well recognized agencies haven't changed as much.

In this paper, I will present a model of universities as evaluators of potential employees. In this model I will show how grade inflation will be the optimal policy of the university seeking to maximize the profits they receive from tuition. The model will also generate grade inflation concentrated primarily in the best universities, while the worst universities will have grades more constant or even deflating.

The basic model is as follows. An employer would like to hire productive workers but not hire unproductive workers. Since a potential worker doesn't have any way to credibly signal their productivity on their own, they must attend a university to be evaluated. The university using assignments, projects, exams, etc. evaluates the worker and provides a grade that employers can observe. The university can commit to a grading scheme and does all this in exchange for a tuition fee from the worker. I characterize the optimal grading scheme and fee for the university.

I then extend the model to a dynamic setting where there are new workers enrolling every period. What's more, the university is not able to perfectly observe the worker's productivity. The university must now try to get workers hired so they can charge a large fee, while also build a reputation for being able to predict worker productivity well. I find pooling equilibria where all universities use the same grading scheme (conditional on their reputation), but based on the universities success rate at predicting worker success the good universities still gain reputation and the poor universities lose reputation. I also find separating equilibria. In these the good universities must first have a very strict grading scheme until they adequatly prove that they can predict worker productivity. Then they can employ a very lenient grading scheme and reap the higher profits from their good reputation. The bad universities employ a moderate grading scheme that is constant over time.

Finally, I show a version with ex ante heterogeneous potential employees. I show that the observed phenomenon in the data are not likely to be driven by the good schools imply getting higher and higher quality students.

2 Grading Schemes to Signal Worker Quality

There are three players in the game. There's the worker, the employer and the university. The worker is one of two types: either a porductive worker (high type), or unproductive worker (low type). The probablility that the worker is high type will be denoted by μ. The employer would like to hire a productive worker, and not hire an unproductive worker. However, the employer cannot observe the worker's type, and the worker doesn't have any creditable way to reveal their type. So, the worker pays the university to evaluate them and publish a rating. The worker pays a fee to the university, the university observes a signal of the worker's quality, and sends a message to the employer.

2.1 Setup

The timing of the game is as follows. First nature selects the worker's type, $\omega \in \{H, L\}$, but this is not observed by any player. Next, the university chooses a grading scheme and a fee. The grading scheme is a mapping from worker quality signals they might observe into messages they might send to the employer. The university can use any potentially mixed strategy on an arbitrary message space. The worker chooses whether or not to enroll in the university If the worker chooses to enroll in the university, then thier quality signal is realized and message is sent according to the grading scheme specified. After recieving the message, the employer chooses whether or not to hire the worker. The employer gets a payoff of one if they hire a productive worker and a payoff of $-\xi$ if they hire an unproductive worker. Thus the employer will choose to hire the worker if they believe the probability the worker is a high type is at least $\bar{\mu} = \frac{\xi}{1+\xi}$. Assume ξ is large enough that $\bar{\mu} > \mu$. So without any additional information, the employer would choose not to hire the worker. The worker recieves a wage of one if they are hired and a wage of zero if they are not hired.

If the university's signal was able to perfectly reveal worker type then this would be the most standard of Bayesain Persuasion games. Denote the precision of the university's signal, referred to as the university's quality, as $\theta \in [0, 1]$. Upon observing the universiy's signal, the posterior belief about the worker's type would be equal to the following.

$$\mu' = \begin{cases} \theta + (1-\theta)\mu & \text{w/ prob. } \mu \\ (1-\theta)\mu & \text{w/ prob. } 1-\mu \end{cases} \tag{1}$$

If $\theta = 1$, the signal is perfectly informative of the worker's type and the posterior will equal 0 or 1. If $\theta = 0$, the signal contains no information and the posterior will be equal to the prior, $\mu' = \mu$. Since the worker gets a payoff of zero if they don't attend the university, the university can set their fee equal to the expected payoff they can get for a student that enrolls. μ' isn't the signal the university receives. It is the posteriors that are induced after observing the signal. Since the distribution of posteriors averages out to the prior (is Bayes plausible), we know that there exists some signal that would induce this distribution of posteriors (see Kamenica and Gentzkow (2011)). It is more convienient to work directly with the posteriors than the signal itself. So I will leave the signal unspecified.

The object of interest is the optimal grading scheme for the university. I am going to charactarize this in the context of perfect Bayesian equilibrium.

2.2 Imperfect Signals

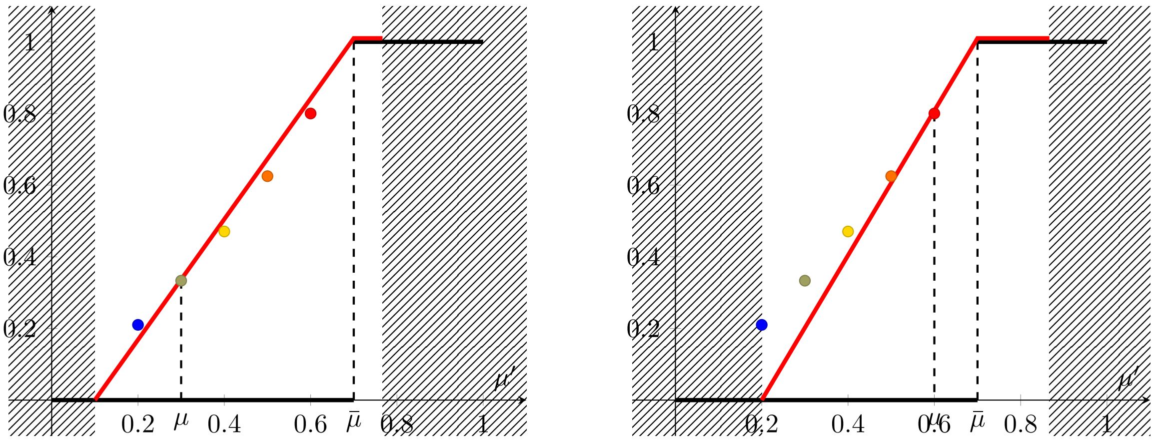
First take $\theta = 1$. This problem is just like the example in the Kamenica and Gentzkow (2011). The university's payoff is equal to the fee they can charge the worker. Therefore, the university would like to choose a grading scheme to maximize the enrolled worker's expected payoff, then charge a fee equal to that expected payoff. There is a simple grading scheme to achieve this. If the university sees that the worker is good, they will pass them. If the university sees that the worker is bad, they will randomize by sometimes passing and sometimes failing them. The university will pass as many workers as possible will still making sure that everyone they pass gets the job. This means the university increases the fraction of bad workers they pass until the employers posterior after observing a pass is exactly equal to $\bar{\mu}$. This gives the usual concavification payoff. The fee they can charge is then equal to $v(\mu) = \frac{\mu}{\bar{\mu}}$.

With $\theta < 1$ the optimal grading scheme is very similar as long as

$$\theta \geq \theta_{min} = \frac{\bar{\mu} - \mu}{1 - \mu}. \tag{2}$$

A grading scheme can induce any distribution of posteriors that second order stochastically dominates the distribution of posteriors obtained by fully revealing their information (see Kolotilin et al. (2017)). When the university can learn perfectly what the worker's type is, the full information distribution of posteriors puts all weight on zero or one. This distribution is second order stochastically dominated by any distribution of beliefs that has the same mean. This is why Kamenica and Gentzkow (2011) has only the condition that the posteriors must have a mean that matches the prior. Here, the full information distribution of posteriors is $\theta + (1 - \theta)\mu$ or $(1 - \theta)\mu$ with frequencies μ and $1 - \mu$ respectively. The university choosing a grading scheme is equivalent to choosing a distribution of posteriors that second order stochastically dominates that full information distribution. Since the full information distribution has all its mass on two points, the second order stochastic dominance condition is simple. A distribution with the same mean will second order stochastic dominate the full information distribution if and only if its support is contained in the interval $[(1 - \theta)\mu, \theta + (1 - \theta)\mu]$. That is, the imperfect information on the part of the university simply puts bounds on the distribution of posteriors they can induce. The only addition requirement beyond beliefs being a martingale, is that the university can not convince the employer of the state beyond their own level of conviction.

The optimal grading scheme will have the same general form as before. The university will pass the worker whenever they get a good signal, and when the university gets a bad signal that will randomize between passing and failing the worker. They will pass just enough bad workers so that the posterior of the employer after observing a pass is equal to $\bar{\mu}$. Call the frequency with which they can pass workers after



(a) $\mu = .3$

(b) $\mu = .6$

Figure 1: Persuasion with imperfect information ($\theta = \frac{2}{3}$).

getting a bad signal $\pi(\theta)$. First see that the posterior belief of the employer after observing a pass is

$$\frac{\mu}{\mu + (1-\mu)\pi}(\theta + (1-\theta)\mu) + \left(1 - \frac{\mu}{\mu + (1-\mu)\pi}\right)(1-\theta)\mu. \tag{3}$$

The first term of this equation is the updated probability that the university got a good signal times the probability the worker is good conditional on the university getting a good signal. The second term is the updated probability that the university got a bad signal times the probability the worker is good conditional on the university getting a bad signal. To get the optimal frequency with which the university passes bad workers, you set this equal to $\bar{\mu}$ and solve for π.

$$\pi^*(\mu) = \frac{\mu}{1-\mu}\left(\frac{\theta}{\bar{\mu} - (1-\theta)\mu} - 1\right) \tag{4}$$

When $\theta = 1$, this is the usual equation.

$$\pi^*(\mu) = \frac{\mu}{1-\mu}\frac{1-\bar{\mu}}{\bar{\mu}} \tag{5}$$

When $\theta = \frac{\bar{\mu}-\mu}{1-\mu}$, the university is not able to pass any bad workers and still have their students hired. Then $\pi^*(\mu) = 0$.

The better the university is, the more often they can pass bad workers. Regardless of θ the posterior after a pass is equal to $\bar{\mu}$. The difference is that the posterior after a fail is increasing in θ. The loss in payoff to the university comes from how often a good worker goes to the university and gets misclassified as bad. Each time that type of misclassification happens, that's a worker everyone agrees should get hired but probably doesn't get hired. Additionally, if all those workers were added into the pool of workers that pass then more bad workers could also be added in to keep the poster at $\bar{\mu}$. In the concavification picture, we can see that the loss in payoff comes from the left endpoint moving to the right.

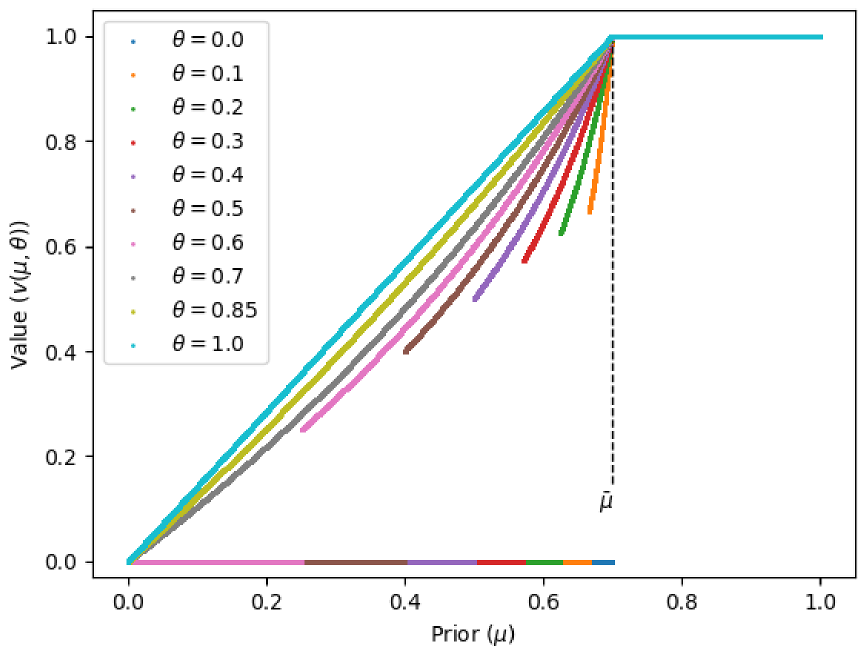


Figure 2: The value to the university as a function of quality.

If $\theta < \frac{\bar{\mu}-\mu}{1-\mu}$, then there is no grading scheme the university is able to employ that will get any of their graduates hired. The university is just viewed by the employer as too low of quality and the employer doesn't trust their judgement enough to hire their graduates. In this case, the university and the worker will both get a payoff of zero regardless of the grading scheme chosen.

2.3 Uncertain School Quality

Suppose that the quality level of the university, θ, is not publicly observed. Given a grading scheme π, the employer's posterior belief depends on the university's quality. Notice from equation (3) that the posterior is a linear function of θ. So, if x is the distribution of θ in the mind of the employer, the posterior belief

$$\frac{\mu}{\mu + (-\mu)\pi}(\mathbb{E}_x\left[\theta\right] + (1-\theta)\mu) + \left(1 - \frac{\mu}{\mu + (1-\mu)\pi}\right)(1 - \mathbb{E}_x\left[\theta\right])\mu. \tag{6}$$

will only depend on the expected value of θ. Also, note that all university types receive a good or bad signal with the same frequency. This means that no updating of x is needed upon observing the university's message (given the grading scheme π).

Now the payoff of all players, including the university, depend only on the expected value of θ and not the actual value. This means the equilibrium will be exactly the same as in the case where the school's quality was known, with that known level being equal to $\mathbb{E}_x[\theta]$. It isn't even necessary for the university to know their own quality level, since their optimal strategy and payoff only depend on the the average quality level.

The average quality level is a public good for all universities now. If at the beginning of the game universities chose a quality level facing some cost function, there would be an underprovision of quality compared to the socially optimal level. This could justify some regulations we see. A university must be accredited before they can give out degrees and credit rating agency must have their methodology filed with the SEC. This underprovision of quality would be expected in any advice giving industry, where the firm cannot know the state perfectly. We see similar legislation arise elsewhere. Financial advisors have a CFA, doctors have a medical license, lawyers pass a bar exam, etc. As we'll see in the next section, the only way such legislation isn't necessary is if the advice giver can establish an individual reputation. However, in many instances this may not be possible. It is difficult for the customer to find the success rate of an individual doctor and so forth.

3 School Reputation

In this section, I will show you how the university can establish an individual reputation to circumvent the public goods problem. We will need to add some dynamics to the model to make this possible. In a one period model, there is nothing a high quality university can do. There is no time for the employers to learn of the university's quality by their success rate. Also, there cannot be a seperating equilibrium where high and low type universities use different grading schemes to reveal themselves. This is because the type, θ, is not directly payoff relevent to the university. When a bad university plays the same action as a good university, they get the same payoff. There is no way to distinguish. In a model like Spence (1973) a seperating equilibrium arises because the different types of students have different costs for attending school. Here, the grading scheme is costless for universities of all types.

If we make this a dynamic game where new workers come to the university every period, both claims above are reversed. Over time, the employer can learn the university's quality by observing their success rate at predicting which workers are good. This learning is hindered somewhat by the fact that universities are not perfectly revealing everything they know. Also, there can be seperating equilibria where high and low quality universities choose different grading schemes. I will explain each of these in turn.

3.1 Pooling Equilibria

I will set up the dynamic game here. It is the same as the one period game except that new workers will arrive every period. I will first characterize the pooling equilibria where high and low type universities use the same grading scheme. Even though the strategies played are the same, employers still learn the university's type through observing success rates over time. Thus, the universities are still able to build a good reputation. Then, I will characterize separating equilibria.

3.1.1 Setup

At the beginning of the game the university's type, θ, is drawn and remains fixed throughout the game. This type is unobserved by the employer. The employer's prior over this type is x_0.

In each period, the university chooses a grading scheme and a fee. Then a worker with productivity $\omega \in \{0, 1\}$ (μ is the probability that $\omega = 1$) chooses whether or not to enroll in the university. If the worker is enrolled, the university sees a signal of the worker's quality (the same way as before) and sends a message to the employer according to their grading scheme. The employer sees the message and chooses whether or not to hire the worker. Then, the worker's productivity (ω) is observed and all players receive their period payoffs. The employer gets a payoff of 1 for hiring a productive worker, $-\xi$ for hiring an unproductive worker, and 0 for not hiring a worker. The university's payoff is equal to the fee they choose if the worker enrolls. The worker gets a payoff of 1 if they are hired and 0 if they are not hired minus the fee they pay to the university if they enroll.

The employer then updates their beliefs about the university's quality. In the next period, another worker is independently drawn and the same game is played again. There is a discount factor of δ between periods.

It isn't important in this section whether the worker or the university know θ. All university types receive a good signal with the same frequency. This means that given the grading scheme, the worker's probability of passing and getting the job doesn't depend on θ. So the worker doesn't care about going to a good university, they only care about going to a university with a good reputation. Also, in this section we are looking at pooling equilibria. You can think of them as pooling equilibria of the game when the university does know their own type, or they would be the only equilibria of the game when the university doesn't know their own type.

3.1.2 Equilibria

The first obvious equilibrium is simply the repeated Nash. The university could play the grading scheme corresponding to

$$\pi^*(\mu, x) = \frac{\mu}{1-\mu}\left(\frac{\mathbb{E}_x[\theta]}{\bar{\mu} - (1 - \mathbb{E}_x[\theta])\mu} - 1\right). \tag{7}$$

Even though the universities are playing the same strategies and it is just the repeated static Nash, there is still learning and reputation. Consider the probabilities of each possible outcome of a period respectively: university says the worker is good and they actually are good, university says good but the worker is bad, university says bad and the worker is bad, and university says bad but their actually good.

$$p^{gg}(\pi, \theta) = \mu\left(\mu + (1-\mu)\pi + (1-\mu)(1-\pi)\theta\right) \tag{8}$$

$$p^{gb}(\pi, \theta) = (1-\mu)\left(\mu + (1-\mu)\pi - \mu(1-\pi)\theta\right) \tag{9}$$

$$p^{bb}(\pi, \theta) = (1-\mu)(1-\pi)(1-(1-\theta)\mu) \tag{10}$$

$$p^{bg}(\pi, \theta) = (1-\mu)(1-\pi)(1-\theta)\mu \tag{11}$$

We can take the derivatives of these probabilities with respect to θ.

$$\frac{\partial p^{gg}}{\partial \theta} = \frac{\partial p^{bb}}{\partial \theta} = -\frac{\partial p^{gb}}{\partial \theta} = -\frac{\partial p^{bg}}{\partial \theta} = \mu(1-\mu)(1-\pi) \tag{12}$$

There are a few things we can notice here. First, the probability of a good message and the probability of a bad message are constants in θ. Thus, the message itself doesn't reveal anything about the univerisity's quality. The learning only happends after the worker's type is revealed. Next see that p^{gg} and p^{bb} are increasing functions of θ while p^{gb} and p^{bg} are decreasing functions of θ. Regardless of the grading scheme, $\pi \in [0, 1)$, the good universities are more likely to correctly identify the productivity of the worker. However as π increases, the dependence on θ goes to zero. The more leanient the grades are, the harder it is to learn not just of the worker quality but also of the university quality.

From this, we can derive the posterior beliefs about university quality after each period outcome. Here x_i denotes the probability that the university is type θ_i. After the university correctly identifies a good worker:

$$x_i^{gg} = x_i \frac{\mu + (1-\mu)\pi + (1-\mu)(1-\pi)\theta_i}{\mu(1-\mu)\pi + (1-\mu)(1-\pi)\mathbb{E}_x[\theta]}. \tag{13}$$

After claiming a worker is good they are actually bad:

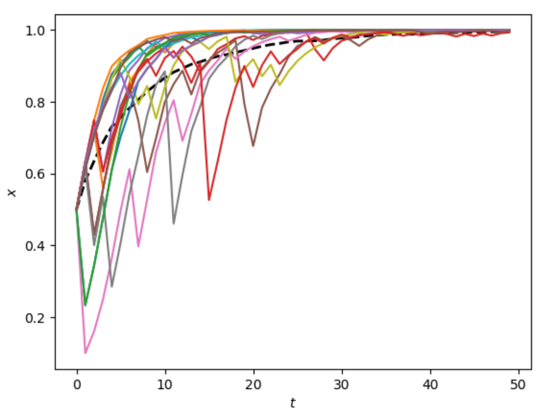
$$x_i^{gb} = x_i \frac{\mu + (1-\mu)\pi - \mu(1-\pi)\theta_i}{\mu + (1-\mu)\pi - \mu(1-\pi)\mathbb{E}_x[\theta]}. \tag{14}$$

After correctly reporting a bad worker:

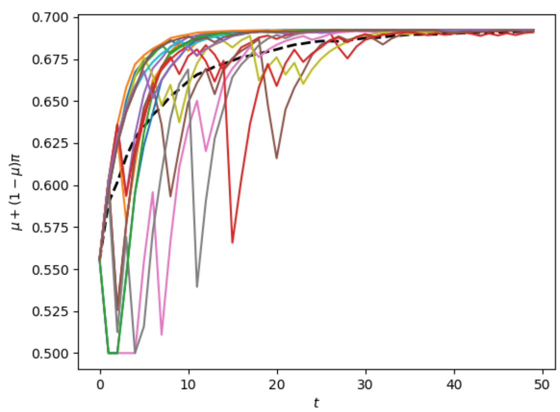
$$x_i^{bb} = x_i \frac{1-(1-\theta_i)\mu}{1-(1-\mathbb{E}_x[\theta])\mu} \quad \forall \pi^x \neq 1. \tag{15}$$

After incorrectly claiming a worker is bad:

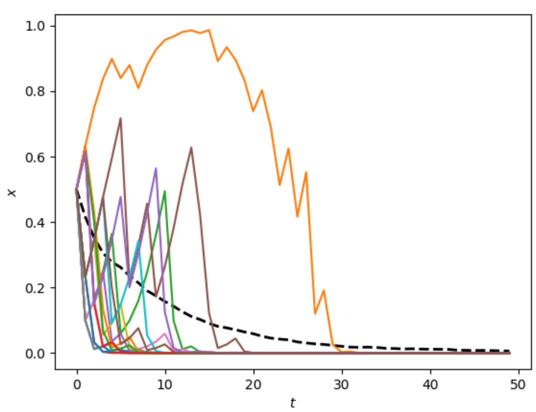
$$x_i^{bg} = x_i \frac{1-\theta_i}{1-\mathbb{E}_x[\theta]} \quad \forall \pi^x \neq 1 \text{ and } \theta \neq 1. \tag{16}$$



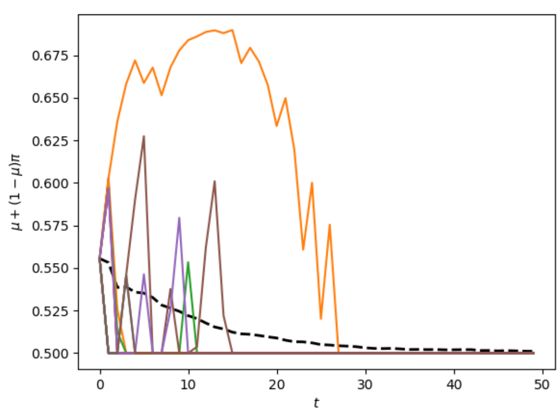
(a) Reputation of the high type over time.



(b) Fraction of students passed over time by the high type.



(c) Reputation of the low type over time.



(d) Fraction of students passed over time by the low type.

Figure 3: Simulations of reputation and pass rate at the university.

We get that the universities are building up their reputation over time. As their reputation grows, the university experiences grade inflation. The bad universities have reputation levels falling over time. At first grades fall too, but soon the university hits the lower bound of reputation where they aren't passing any bad students. For all reputation levels below that, the grading scheme is constant. GPAs are fairly constant over time at the bad universities but rising at the good universities, consistent with the data.

When the university has a good reputation and employs a lenient grading scheme, learning slows down. The grades are less informative about worker productivity, so getting the grade "right" or "wrong" doesn't mean as much about the university's quality. Conversely when the reputation is low, the grading scheme is very rigorous. This means that if a worker passes or fails it is because the university thinks they are good or bad respectively. Any worker incorrectly identified is due to a mistake by the university. Mistakes are then very informative about the university quality. So, a bad university that lucks into a high reputation can maintain that reputation for a while and only slowly be revealed. On the other hand a good university that lucks into a bad reputation is able to turn things around very quickly and bring their reputation back up.

3.2 Seperating Equilibrium

Unlike the one period model, there can be separating equilibria in the multi-period model. In this section, suppose there are only two quality levels for the university, $\theta \in \{\theta_H, \theta_L\}$. Separating equilibria can only exist if θ_L is sufficiently high. If $\theta_L \leq \frac{\bar{\mu}-\mu}{1-\mu}$, then a university revealed as the low type will get zero profits forever. Assuming a university can't have negative fees, the low type university will always be better off pretending to be a high type.

Call π_L the optimal grading scheme for a university known to be low type, $\theta_L \geq \frac{\bar{\mu}-\mu}{1-\mu}$.

$$\pi_L = \frac{\mu}{1-\mu}\left(\frac{\theta_L}{\bar{\mu}-(1-\theta_L)\mu} - 1\right) \tag{17}$$

In the separating equilibria, the low type universities will play π_L every period. The high type universities will play a history dependent grading scheme, $\pi(h^t)$. If the university were to play any off-path action, the employer would believe them to be the low type. I will now describe the conditions on $\pi(h^t)$ required to make this constitute an equilibrium.

First, we need the low type university not to want to pretend to be the high type for any number of periods. This requires that

$$\pi_L \left(\frac{1-\delta^\tau}{1-\delta} \right) \geq \mathbb{E}\left[\sum_{t=0}^{\tau-1} \delta^t \pi(h^t) \mid \theta = \theta_L \right] \qquad \forall \tau = 1, 2, \dots \tag{18}$$

It needs to be that the for any number of periods, from the start of the game, the low type would be better off playing π_L than playing the high type's strategy in expectation. Taking $\tau = 1$ this necessarily implies that the high type will play a more revealing grading scheme at first than the low type, $\pi(h^0) < \pi_L$. In any separating equilibrium, the high type will be initially playing a more stringent (revealing) grading scheme.

The other necessary condition is that the high type would be better of playing $\pi(h^t)$ than playing π_L for the rest of the game after any history. This will require the following.

$$\mathbb{E}\left[\sum_{t=\tau}^{T} \delta^{t-\tau} \pi(h^t) \mid \theta = \theta_H, h^\tau \right] \geq \pi_L \left(\frac{1-\delta^{T-\tau}}{1-\delta} \right) \qquad \forall \tau, h^\tau \tag{19}$$

At any point in time, τ, no matter what has happened up until that point, it must be better for the the high type to stay the course by playing $\pi(h^t)$ than switch to π_L. Since we know that the high type is playing a lower π in the first periods than the low type, for this condition to hold when $\tau = 0$ it must be that the high type is playing a higher π in the later periods. We can already see that a separating equilibrium will have grade inflation for the good universities but a constant grading scheme for the bad universities.

If we take $\tau = T$ in equation (18) and we take $\tau = 0$ in equation (19), the two equations seem to nearly make a contradiction.

$$\mathbb{E}\left[\sum_{t=0}^{T} \delta^t \pi(h^t) \mid \theta_H \right] \geq \pi_L \frac{1-\delta^T}{1-\delta} \geq \mathbb{E}\left[\sum_{t=0}^{T} \delta^t \pi(h^t) \mid \theta_L \right] \tag{20}$$

In fact, this can still be consistent (even if the inequalities were strict). The separating equilibria still look a little different from those in other models though. There will never be an equilibrium like, "the high type must play a low π for two periods and then they can play π_H forever," or anything similar. Anything such rule for $\pi(h^t)$ will have the same problem that the one period model had. It will always have the same payoff for both types. If the high type finds it profitable to do that strategy, the low type will also. With any rule like that, the left most term in inequality (20) and the right most term in inequality (20) will be equal. The only difference between those two terms in the inequality is in the expectation. We saw earlier that the probability of each outcome (a message and worker productivity realization together) is dependent on the university quality, even though the message distribution isn't. Thus the only way to get separation between the two terms of the inequality is to make $\pi(h^t)$ depend on both the message distributions and the outcomes of previous periods. Instead of the high type having to play a low π for two periods, they may need to play a low π until correctly identifying two workers' productivity for example. The correct identification is more likely for the high type with any given grading scheme. Thus, the low type university would need to play the low π for longer on average. This is the only way to get different "costs" that depend on the university's quality.

We also know that neither type will want to deviate to anything other than π_L or $\pi(h^t)$. At any other deviation, the employer assumes the university is low quality. We have already established that π_L is the best thing the university can play if they are known to be low quality.

Together with the grading scheme being such that workers that pass will get hired,

$$\pi(h^t) \leq \frac{\mu}{1-\mu}\left(\frac{\theta_H}{\bar{\mu}-(1-\theta_H)\mu}-1\right), \tag{21}$$

inequalities (18) and (19) are sufficient conditions for $\pi(h^t)$ to be the strategy of the high type in a separating equilibrium where the low type plays π_L.

3.2.1 Example

In this section, I will show an example of such a separating equilibrium for given parameter values. Consider a two period model. Let the worker be either productivity level with equal probability, $\mu = \frac{1}{2}$. Also let $\xi = 3$ so the required posterior for the worker to get hired is $\bar{\mu} = \frac{3}{4}$. Then the maximal grading scheme for a university of type θ in equation (4) simplifies.

$$\pi^\theta = \frac{\theta - \frac{1}{2}}{\theta + \frac{1}{2}} \tag{22}$$

Suppose also that the high type university is able to perfectly predict the worker's productivity, $\theta_H = 1$.

In equilibrium the low type will play π_L both periods. The high type will play $\pi = 0$ in the first period. If the university playing $\pi = 0$ in the first period correctly identifies the worker productivity, then in the second period they will play $\pi = \pi_H$. If the university is incorrect in the first period, in the second period they will play π_L. The low type is able to employ some persuasion by passing some bad workers each period. The high type will take a lower payoff in the first period by employing no persuasion. The high type simply reveals the worker's productivity perfectly without passing any bad workers. Then, after correctly identifying the first worker's productivity and proving they are a high type, the university can use a large amount of persuasion in the second period. The low type will unwilling to copy the high type's action because there is a chance they would misidentify the period one worker's productivity and be revealed as a low type.

First, we'll look at the condition to ensure the high type doesn't deviate to the low type's strategy.

$$\mu + \delta\left(\mu + (1-\mu)\pi_H\right) \geq (1+\delta)\left(\mu + (1-\mu)\pi_L\right) \tag{23}$$

This will hold as long as the payoff to the low type is sufficiently low.

$$\pi_L \leq \frac{\delta}{1+\delta}\pi_H \tag{24}$$

Using $\theta_H = 1$ and $\delta = 1$, this becomes a condition on θ_L.

$$\frac{\theta_L - \frac{1}{2}}{\theta_L + \frac{1}{2}} \leq \frac{1}{6} \tag{25}$$

$$\theta_L \leq .7 \tag{26}$$

So this type of equilibrium is can only be supported if θ_L is low enough.

Now we need to make sure that the low type doesn't want to deviate to playing the high type's strategy. This requires that playing π_L in both periods is better than playing $\pi = 0$ in the first period and playing π_H in the second period if the first period identification was correct and playing π_L in the second period if the first period identification was incorrect. Call $p^{m\omega}$ the probability of sending message m and the worker's type being ω, with the grading scheme $\pi = 0$ and university quality $\theta = \theta_L$, $p^{m\omega} = p^{m\omega}(0, \theta_L)$. The condition is now the following.

$$\begin{aligned}(1+\delta)\left(\mu + (1-\mu)\pi_L\right) \;\;\geq\;\; \mu \;+\; & \delta\left(p^{gg} + p^{bb}\right)\left(\mu + (1-\mu)\pi_H\right) \\ +\; & \delta\left(p^{gb} + p^{bg}\right)\left(\mu + (1-\mu)\pi_L\right)\end{aligned} \tag{27}$$

This can be simplified to be only the additional gain from each strategy.

$$\pi_L \;\;\geq\;\; \delta\left(p^{gg} + p^{bb}\right)\left(\pi_H - \pi_L\right) \tag{28}$$

The benefit of playing the low type strategy is getting π_L of the bad workers hired in the first period while the high type isn't getting any. The benefit of playing the high type strategy is the extra bad workers you can get hired in the second period, $\pi_H - \pi_L$, if you're able to pass the first period test of correctly identifying a worker. Rearrange the inequality to get the restriction on π_L.

$$\pi_L \;\;\geq\;\; \frac{\delta\left(p^{gg} + p^{bb}\right)}{1 + \delta\left(p^{gg} + p^{bb}\right)}\pi_H \tag{29}$$

Using the probability equations above (equation (8)), we get the following.

$$p^{gg} + p^{bb} = 1 \; - \; 2\mu(1-\mu)(1-\theta) \tag{30}$$

Now we can plug this into our restriction on π_L along with $\theta_H = 1$, $\mu = \frac{1}{2}$, and $\delta = 1$.

$$\pi_L \;\geq\; \frac{1 - \frac{1}{2}(1-\theta_L)}{2 - \frac{1}{2}(1-\theta_L)} \frac{1}{3} \tag{31}$$

Putting in the equation for π_L and solving for θ_L, we get $\theta_L \geq .68$. We find that θ_L needs to be sufficiently high that the low type doesn't want to give up their first period gains for a gamble.

Together with the restriction to keep the high type from deviating, we have now found that if $\pi_L \in [.68, .7]$ this constitutes a separating equilibrium. Observe the grading inflation pattern in this equilibrium. Take $\theta_L = .7$. Exactly one half of the workers have a high productivity each period. The poor quality universities are passing $\frac{7}{12}$ of their workers every period. The bad university lets a few workers skate by when they don't deserve it. The high quality university only passes $\frac{1}{2}$ of the workers in the first period, but in the second period they pass $\frac{2}{3}$ of the workers. At first the high quality university only passes worker that truely deserve it, but after some time they start letting more bad workers skate by than the low quality university did.

4 Heterogeneity

Some will certainly argue that good schools give out better grades because they have better students. This would imply that grade inflation at good schools is being driven by the good schools getting better and better students. First since the employer knows a school is better and will account for that and what workers will attend in equilibrium in forming their posterior beliefs, that really isn't a reason for higher grades. However, let's now extend the model a bit to add in ex ante heterogeneous workers and see some other reasons this won't lead to the grading patterns observed in the data.

Say there are a mass of workers and the employer (and university) have different priors about different workers' productivity even before they attend the university. The workers have different grades from high school, SAT scores, extra curricular activities, recommendations, etc. There is a unit mass of workers and the priors over worker productivity is uniformly distributed on the unit interval, $\mu \nabla U[0,1]$. There will be only one period. The university will choose only one grading scheme, π, and one fee, f. Then each worker with their individual likelihood of being productive (prior, μ) will choose whether or not to enroll in the university. For every worker enrolled at the university, the university will receive a signal of productivity and a message will be sent to the employer according to the grading scheme. Call the set $\mathcal{E} \subset [0,1]$ the set of workers that choose to enroll in the university. The university's payoff is then

$$v = \int_0^1 f \mathbf{1}_{\mathcal{E}} \, dx \tag{32}$$

or f times the measure of the set $\mathcal{E}$.

The university's grading scheme will be of the same form as earlier. The university will pass all the workers it receives a good signal about, and also pass π fraction of the workers it receives a bad signal about. Note that given the grading scheme and the fee, if a worker at prior μ finds it profitable to enroll then a worker at prior $\mu' > \mu$ will also find it profitable to enroll (except of course workers with prior $\mu' > \bar{\mu}$ who will choose not to enroll regardless of the fee.) This means that the set of students who choose to enroll will always be a simple interval, $\mathcal{E} = [\hat{\mu}, \bar{\mu}]$ where $\hat{\mu}$ is such that

$$\hat{\mu} + (1 - \hat{\mu})\pi = f. \tag{33}$$

We can think of the university's problem as one of choosing $\hat{\mu}$ then back out π and f from that. π will be the highest π that can still get the $\hat{\mu}$ workers the job after a good signal and f will equal the probability of the $\hat{\mu}$ worker passing.

The value to the university from worker $\hat{\mu}$ is then

$$v(\hat{\mu}) = \hat{\mu} + (1 - \hat{\mu})\pi^*(\hat{\mu}) \tag{34}$$

$$= \frac{\hat{\mu}\mathbb{E}[\theta]}{\bar{\mu} - (1 - \mathbb{E}[\theta])\hat{\mu}}. \tag{35}$$

The overall value to the university is found by then multiplying by the measure of workers that enroll.

$$v = \frac{\hat{\mu}\mathbb{E}[\theta]}{\bar{\mu} - (1 - \mathbb{E}[\theta])\hat{\mu}}(\bar{\mu} - \hat{\mu}) \tag{36}$$

The university will choose $\hat{\mu}$ to maximize v. Differentiate v with respect to $\hat{\mu}$.

$$\frac{\partial v}{\partial \hat{\mu}} = \frac{\bar{\mu}\mathbb{E}[\theta] - 2\mathbb{E}[\theta]\hat{\mu}}{\bar{\mu} - (1 - \mathbb{E}[\theta])\hat{\mu}} + \frac{(1 - \mathbb{E}[\theta])(\bar{\mu} - \hat{\mu})\hat{\mu}\mathbb{E}[\theta]}{(\bar{\mu} - (1 - \mathbb{E}[\theta]))^2} \tag{37}$$

The second derivative is will show that we a finding a local maximum. Set this derivative equal to zero and solve for the optimal $\hat{\mu}$.

$$\hat{\mu}^* = \bar{\mu}\frac{1 - \sqrt{\mathbb{E}[\theta]}}{1 - \mathbb{E}[\theta]} \tag{38}$$

The lower end of workers that enroll is a decreasing function of the university's reputation. If the university is known to be no good ($\mathbb{E}[\theta] = 0$), then they set $\hat{\mu}^* = \bar{\mu}$. So, no workers enroll. As reputation increases, the university lowers $\hat{\mu}^*$. When the university is known to be the most knowledgable ($\mathbb{E}[\theta] = 1$), the lower bound of workers approaches $\hat{\mu}^* = \frac{\bar{\mu}}{2}$. Exactly half of the workers looking to go to school choose to enroll.

We can now back out the optimal grading scheme of the university.

$$\pi^* = \pi_\theta(\hat{\mu}^*) \tag{39}$$

$$= \frac{\hat{\mu}^*}{1 - \hat{\mu}^*} \left(\frac{\mathbb{E}[\theta]}{\bar{\mu} - (1 - \mathbb{E}[\theta])\, \hat{\mu}^*} - 1 \right) \tag{40}$$

Now insert the equation for $\hat{\mu}^*$ we just solved for and simplify.

$$\pi^* = \frac{\sqrt{\mathbb{E}[\theta]} - \mathbb{E}[\theta] - \bar{\mu}\left(1 - \sqrt{\mathbb{E}[\theta]}\right)}{1 - \mathbb{E}[\theta] - \bar{\mu}\left(1 - \sqrt{\mathbb{E}[\theta]}\right)} \tag{41}$$

The optimal π is an increasing function of the university's reputation.

These equations illustrate story of why the good universities experience grade inflation that isn't based on them having better students. We see that $\hat{\mu}^*$ is a decreasing function of $\mathbb{E}[\theta]$ while π^* is an increasing function of $\mathbb{E}[\theta]$. The story this model presents is that back in the day, before Harvard (any high quality university) established their reputation, only the very best students went to Harvard. Also for the students that did go to Harvard, it was very difficutly to pass and get good grades. However, Harvard's reputation grew over time. Now everyone knows that Harvard is excellent. So now more students go to Harvard bringing their average student quality down (but still not nearly all students). Also Harvard can now be more lenient in their grading. It is easier to get a passing grade at Harvard now than it once was.

5 Conclusion

In this paper, I presented a model of universities as being hired by a worker to evaluate them for potential employers. I show how reputation concerns can enter the problem of an information designer with imperfect inforamtion. I believe these concerns can enter into all standard situations of a strategic player sharing advice or information. The model gives an explanation for grade inflation, particularly as concentrated among the best universities.

There are many papers on related topics. I would like to mention only a few that I found essential to this paper. Another paper with a model seeking to explain grade inflation is Boleslavesky and Cotton (2015). In their model grade inflation is driven by the university investing more in the quality of their education. My paper is more after the models of Spence (1973) and various extensions. The role of the university here is to help the employers distinguish the productive and unproductive workers. I use a framework and techniques recent developed by information design papers such as Kamenica and Gentzkow (2011) and Kolotilin et al. (2017). Lastly, my separating equilibrium conditions resemble those that were well studied by Kaya (2009).

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