Does the Distribution of Private Information in Financial Markets Matter?

Isaac Swift

September 22, 2023

Updated version at: Link

Abstract

Holding fixed the total amount of information, which is preferred, a market where one trader has more private information, or a market where two traders each have less private information? Using a Kyle model, I show an equivalence between two informed traders with independent pieces of information and a single informed trader with both pieces of information. However, the equivalence fails when comparing two traders with the opportunity to acquire costly pieces of independent information and a single trader with the opportunity to acquire both pieces of information. Liquidity is a public good that is reduced by the presence of private information. Multiple traders will over acquire information ignoring the negative externalities on other traders through liquidity effects. Hence, a more dispersed distribution of information acquisition opportunities leads to lower liquidity, harming traders, but greater informativeness of prices. These results are exactly opposite to most intuition on insider trading and its regulation.

1 Introduction

Imagine there are many independent pieces of private information regarding the value of an asset. As traders use that information, prices adjust to incorporate the information. The amount of information (among other things) affects liquidity, profitability of traders, and informativeness of asset prices. The question of this paper is, does does the distribution of that information matter? What's the difference to liquidity, profit, or informativeness if just one trader had all those pieces of private information instead of it initially being dispersed among many traders?

I show first that the distribution of information among traders does not matter as long as that information is independent. However, the distribution of opportunities to acquire information does impact markets. As the information acquisition opportunities become more concentrated, prices become less informative and liquidity goes up. This leads to higher profits for traders. The intuition is as follows. When a trader acquires more information, liquidity in the market is reduced. Less liquidity is a negative externality on all traders. Thus, there will be too much information, relative to what maximizes trader profits. This effect is stronger the more dispersed the information acquisition opportunities.

There are countless market microstructure papers showing the effect of private information on liquidity, profitability of traders, and informativeness of asset prices, such as Kyle (1985), Glosten and Milgrom (1985), Swift (2023). Some of these already include information acquisition, Mendelson and Tunca (2004), or multiple informed traders, Foster and Viswanathan (1996), Holden and Subrahmanyan (1992). Some discuss the distribution of information, such as Holden and Subrahmanyan (1992) and Fishman and Hagerty (1992). This papers find the rat race effect of more traders increasing the speed of trade. No paper studies this topic with independent information (where there would be no rat race effect). To be clear, papers may have conditionally independent signals of a value, but they are not completely independent.

Here $N = 2$, but it all works out the same with any N. I do it all in one period because there is no need for dynamics. I use a Kyle model (Kyle (1985)), but in the online appendix I get the same results with a Glosten and Milgrom model (Glosten and Milgrom (1985)).

2 Model

There are two stages. In the first stage, traders can acquire information at a cost. The second stage is a standard Kyle model with multiple informed traders that have independent pieces of information. There is an asset with an unknown common value comprised of two independent components, $v = v_1 + v_2$. The common priors are normally distributed, $v_1 \sim \mathcal{N}(0, \sigma^2_{v_1})$ and $v_2 \sim \mathcal{N}(0, \sigma^2_{v_2})$. In the first stage, a signal can be observed, $s_1 \sim \mathcal{N}(v_1, \sigma^2_{s_1})$ and $s_2 \sim \mathcal{N}(v_1, \sigma^2_{s_2})$. The point is, there are two pieces of information that are both relevant to the value of the asset but are independent of each other. The point of the independence is simply to remove the "rat race" effect already well understood from papers such as Holden and Subrahmanyan (1992). The trader can choose how much information to observe by choosing the precision of the signal, subject to a cost function. We'll assume the cost functions, $c_1(\sigma_{s_1})$ and $c_2(\sigma_{s_2})$, are decreasing, convex, and differentiable.

In the second stage, there is trade. There is a mass of unmodeled liquidity traders that submit an order of $u \sim \mathcal{N}(0, \sigma_u^2)$. There are two strategic, potentially informed, traders, $i \in \{1, 2\}$. The strategic traders using their own private information (s_1, s_2, both, or neither), submit a marker order $x_i \in \mathbb{R}$ to maximize their expected payoff, π_i.

$$\mathbb{E}[\pi_i] = \mathbb{E}\left[(v - p)x_i\right] \tag{1}$$

The competitive market maker observes the total demand, $y = x_1 + x_2 + u$, and sets the price equal to the the expected common value of the asset conditional on the information contained in y.

$$p = \mathbb{E}[v|y] \tag{2}$$

The goal is to compare the situation where trader 1 can observe s_1 and trader 2 can observe s_2 to the situation where trader 1 observes s_1 and s_2 and trader 2 observes no private information.

Proposition 1. *The in the second stage, distribution of information doesn't matter. More precisely, holding fixed σ_{s_1} and σ_{s_2}, whether trader 1 observes s_1 and trader 2 observes s_2 or trader 1 observes s_1 and s_2 while trader 2 observes nothing, the prices, liquidity, and total payoffs are the same.*

Proof. First, consider the case where trader 1 observes s_1 and trader 2 observes s_2. Supposing the other informed trader and the market maker play linear strategies, $x_2 = \beta_2 s_2$ and $p = \lambda y$, trader 1's problem becomes,

$$\max_{x_1} \; \mathbb{E}\left[(v_1 + v_2 - \lambda(x_1 + \beta_2 s_2 + u))x_1\right] \tag{3}$$

Maximizing gives the optimal strategy for the informed traders. $x_1 = \frac{s_1}{2\lambda}\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2+\sigma_{s_1}^2}$ and $x_2 = \frac{s_2}{2\lambda}\frac{\sigma_{v_2}^2}{\sigma_{v_2}^2+\sigma_{s_2}^2}$, giving $y = \frac{1}{2\lambda}(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2+\sigma_{s_1}^2}s_1 + \frac{\sigma_{v_2}^2}{\sigma_{v_2}^2+\sigma_{s_2}^2}s_2) + u$. The market maker's price is the expected value conditional on observing y, which is linear by the usual argument.

$$p^* = \frac{\beta_1\sigma_{v_1}^2 + \beta_2\sigma_{v_2}^2}{\beta_1^2(\sigma_{v_1}^2 + \sigma_{s_1}^2) + \beta_2^2(\sigma_{v_2}^2 + \sigma_{s_2}^2) + \sigma_u^2}y \tag{4}$$

Now consider the case where trader 1 observes s_1 and s_2 and trader 2 observes nothing. Supposing the other informed trader orders nothing and the market maker plays a linear strategy, $x_2 = 0$ and $p = \lambda y$, trader 1's problem becomes,

$$\max_{x_1} \; \mathbb{E}\left[(v_1 + v_2 - \lambda(x_1 + u))x_1\right] \tag{5}$$

Maximizing gives the optimal strategy for the informed traders. $x_1 = \frac{1}{2\lambda}(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2+\sigma_{s_1}^2}s_1 + \frac{\sigma_{v_2}^2}{\sigma_{v_2}^2+\sigma_{s_2}^2}s_2)$ and $x_2 = 0$, giving $y = \frac{1}{2\lambda}(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2+\sigma_{s_1}^2}s_1 + \frac{\sigma_{v_2}^2}{\sigma_{v_2}^2+\sigma_{s_2}^2}s_2) + u$. Since y is the same, this scenario is indistinguishable to the market maker and they must also set the same price. Thus, the price, liquidity, and total demand are unchanged as well as total profits to strategic traders, the market maker, and liquidity traders. □

While we just saw the distribution of information doesn't matter, the distribution of opportunities to acquire information does matter.

Proposition 2. *Compared to the situation where a single trader can acquire both pieces of information, if trader one can acquire s_1 and trader 2 can acquire s_2, liquidity will be lower, informed trader profits will be lower, and prices will be more informative.*

Proof. First, let's solve the situation with a single informed trader. Their expected profit is

$$\mathbb{E}[(v_1 + v_2 - \lambda(x^* + u))x^*] \tag{6}$$

with x^* being the optimal value found above. After observing s_1 and s_2, the expectation can be evaluated as the following.

$$\left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{s_1}^2}s_1 + \frac{\sigma_{v_2}^2}{\sigma_{v_2}^2 + \sigma_{s_2}^2}s_2\right)x^* - \lambda x^{*2} = \frac{1}{4\lambda}\left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{s_1}^2}s_1 + \frac{\sigma_{v_2}^2}{\sigma_{v_2}^2 + \sigma_{s_2}^2}s_2\right)^2 \tag{7}$$

Now we want to find the expected value of this given chosen values of σ_{s_1} and σ_{s_2}. Since s_1 and s_2 are both mean zero, this equation becomes a variance.

$$\begin{aligned}\frac{1}{4\lambda}V\left[\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{s_1}^2}s_1 + \frac{\sigma_{v_2}^2}{\sigma_{v_2}^2 + \sigma_{s_2}^2}s_2\right] &= \frac{1}{4\lambda}\left(\left(\frac{\sigma_{v_1}^2}{\sigma_{v_1}^2 + \sigma_{s_1}^2}\right)^2(\sigma_{v_1}^2 + \sigma_{s_1}^2) + \left(\frac{\sigma_{v_2}^2}{\sigma_{v_2}^2 + \sigma_{s_2}^2}\right)^2(\sigma_{v_2}^2 + \sigma_{s_2}^2)\right) \\ &= \frac{1}{4\lambda}\left(\frac{\sigma_{v_1}^4}{\sigma_{v_1}^2 + \sigma_{s_1}^2} + \frac{\sigma_{v_2}^4}{\sigma_{v_2}^2 + \sigma_{s_2}^2}\right)\end{aligned}$$

Calling $w_1(\sigma_{s_1}) = \frac{\sigma_{v_1}^4}{\sigma_{v_1}^2+\sigma_{s_1}^2}$, we can now write the trader's maximization problem.

$$\pi = \max_{\sigma_{s_1},\sigma_{s_2}} \frac{1}{4\lambda}\left(w_1(\sigma_{s_1}) + w_2(\sigma_{s_2})\right) - c_1(\sigma_{s_1}) - c_2(\sigma_{s_2}) \tag{8}$$

The first order condition is,

$$-\frac{\lambda'}{4\lambda^2}(w_1 + w_2) + \frac{1}{4\lambda}w_1' - c_1' = 0 \tag{9}$$

While the same condition in the case of two informed traders would be

$$-\frac{\lambda'}{4\lambda^2}w_1 + \frac{1}{4\lambda}w_1' - c_1' = 0. \tag{10}$$

Notice that λ, w_1, and w_2 are positive while λ', w_1', and c_1' are negative. So, the derivative starts positive and crosses to the negative to the right. The condition for the single informed trader has a strictly positive term added on, and will thus cross zero at a point farther to the right. This means a higher value for σ_{s_1}. A higher σ_{s_1} corresponds to less information. Liquidity in the model is represented by $\frac{1}{\lambda}$ and lambda is a decreasing function of σ_{s_1}. So, the single informed trader situation has higher liquidity.

We can also tell that total profit to the informed trader(s) is higher in the single informed trader case because they could have chosen the same amount of information as the equilibrium with two informed traders and made the same profit, but they didn't. $\square$

While the noise traders aren't modeled explicitly, under any reasonable model the lower liquidity is going to be detrimental to them.

3 Conclusion

Consider the regulation of insider trading. It has been argued that allowing a few well-informed insiders to trade, will crowd out a large number of analysts and other traders, see Fishman and Hagerty (1992). If the total amount of information is held constant, in what way is the many analysts with a small amount of information preferable to a small number of insiders with a lot of information? Such questions haven't been adequately studied. Much analysis of insider trading regulation argues that the regulation will lead to less information in prices, but will improve trader welfare by helping liquidity, removing the "insider trading tax", see Manne (1966), Leland (1992), or Bainbridge (2001). I show in this paper that that the distributional effects of information are nearly completely the opposite.

References

Bainbridge, S. M. (2001). The law and economics of insider trading: A comprehensive primer. *Working Paper, University of California, Los Angeles.*

Fishman, M. J. and Hagerty, K. M. (1992). Insider trading and the efficiency of stock prices. *The RAND Journal of Economics*, 23(1):106–122.

Foster, F. D. and Viswanathan, S. (1996). Strategic trading when agents forecast the forecasts of others. *The Journal of Finance*, 51(4):1437–1478.

Glosten, L. R. and Milgrom, P. R. (1985). Bid, ask and transaction prices in a specialist market with heterogeneously informed traders. *Journal of Financial Economics*, 14(1):71–100.

Holden, C. W. and Subrahmanyan, A. (1992). Long-lived private information and imperfect competition. *The Journal of Finance*, 47(1):247–270.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica*, 53(6):1315–1336.

Leland, H. E. (1992). Insider trading: Should it be prohibited? *The Journal of Political Economy*, 100(4):859–887.

Manne, H. G. (1966). *Insider Trading and the Stock Market.* Free Press, New York.

Mendelson, H. and Tunca, T. I. (2004). Strategic trading, liquidity, and information acquisition. *The Review of Financial Studies*, 17(2):295–337.

Swift, I. (2023). Dynamics of price discovery. *Working Paper.*